Koide Formula for Neutrino Masses

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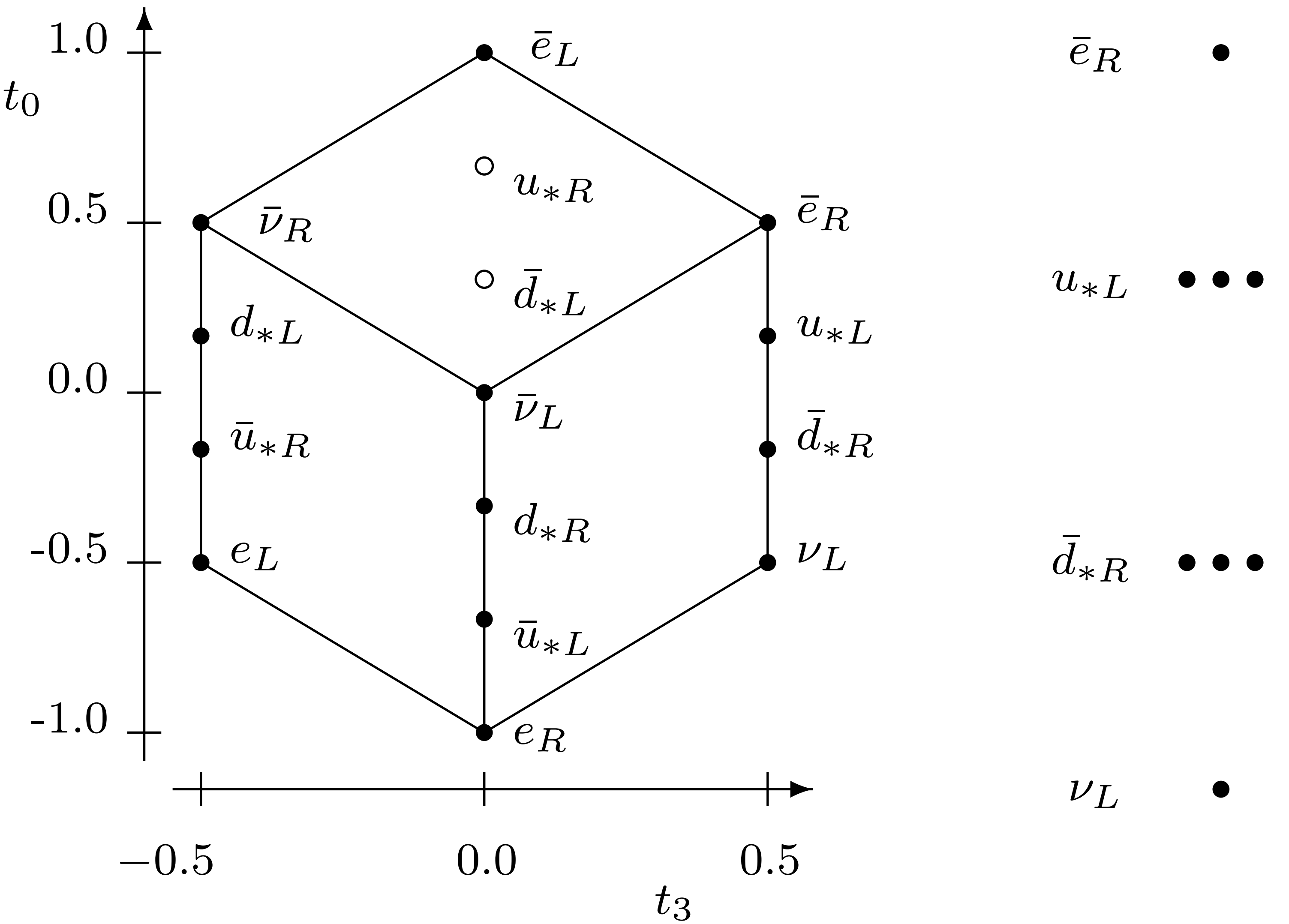
www.PhysicsForums.com

The ν_R is not shown for clarity.



Hermitian circulant matrix has μ, η, δ real:

$$\Gamma(\mu,\eta,\delta) = \mu \begin{pmatrix} 1 & \eta e^{+i\delta} & \eta e^{-i\delta} \\ \eta e^{-i\delta} & 1 & \eta e^{+i\delta} \\ \eta e^{+i\delta} & \eta e^{-i\delta} & 1 \end{pmatrix}.$$

The eigenvectors:

$$|n\rangle = \begin{pmatrix} 1 \\ e^{+2in\pi/3} \\ e^{-2in\pi/3} \end{pmatrix} \quad n = 1, 2, 3$$

The eigenvalues:

$$\lambda_n = \mu(1 + 2\eta\cos(\delta + 2n\pi/3))$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 3\mu$$

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 3\mu^2(1 + 2\eta^2)$$

so

$$\eta^2 = \tfrac{3}{2}\tfrac{\lambda_1^2+\lambda_2^2+\lambda_3^2}{(\lambda_1+\lambda_2+\lambda_3)^2} - \tfrac{1}{2}$$

letting

$$\lambda_1 = \sqrt{m_e}, \qquad \lambda_2 = \sqrt{m_\mu}, \qquad \lambda_3 = \sqrt{m_\tau}$$

gives

$$\eta_1^2 = 0.500003(23),$$

$$\delta_1 = 0.2222220(19)$$

Koide's charged lepton mass formula:

$$\frac{(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau})^2}{m_e+m_\mu+m_\tau} = \frac{3}{2}.$$

can be put to neutrinos if a square root is negative:

$$\begin{aligned} m_{\nu 1} &= 0.000388(46)\ \text{eV}, \\ m_{\nu 2} &= 0.00895(17)\ \text{eV}, \\ m_{\nu 3} &= 0.0507(30)\ \text{eV}. \end{aligned}$$

$$\frac{(-\sqrt{m_{\nu 1}}+\sqrt{m_{\nu 2}}+\sqrt{m_{\nu 3}})^2}{m_{\nu 1}+m_{\nu 2}+m_{\nu 3}} = \frac{3}{2}$$

Koide explains mass generational hierarchy for leptons.

Hierarchy between electron and neutrino much bigger.

Let μ_1, η_1, and δ_1 apply to the charged leptons, and μ_0, η_0, and δ_0 apply to the neutrinos.

Since $\eta_1^2 = 0.5$ within experimental error, we suppose that it is exact, and also that $\eta_0 = \eta_1$. Applying the restrictions to the measured neutrino mass differences we find:

$$\begin{aligned}\delta_1 &= \delta_0 - \tfrac{\pi}{12}1.008(80),\\ \mu_1/\mu_0 &= (2.9999(71))^{11}.\end{aligned}$$

If exact, can give precision estimates of neutrino masses:

$$
\begin{aligned}
m_{\nu 1} &= 0.000383462480(38) & \text{eV}, \\
m_{\nu 2} &= 0.00891348724(79) & \text{eV}, \\
m_{\nu 3} &= 0.0507118044(45) & \text{eV}, \\
m_{\nu 2}^2 - m_{\nu 1}^2 &= 7.930321129(141) \times 10^{-5} & \text{eV}^2, \\
m_{\nu 3}^2 - m_{\nu 2}^2 &= 2.49223685(44) \times 10^{-3} & \text{eV}^2,
\end{aligned}
$$

Various neutrino mass calculations, in eV:

m_1	.0000	.0000	.0000	**.0004**	.0030	.0041	.0049	.0170
m_2	.0083	.0520	.0089	**.0090**	.0088	.0097	.0098	.0170
m_3	.0500	.0530	.0508	**.0508**	.0500	.0510	.0500	.0530

hep-ph/0308097, hep-ph/0503159, hep-ph/0601098, **this**, hep-ph/0303256, hep-ph/0601104, hep-ph/0403077, hep-ph/0512009

Since δ is arbitrary modulo $2n\pi/3$, and since we can put $\eta_0 = -\eta_1$, we can suppose (to first order):

	electron	neutrino
η	$+\sqrt{0.5}$	$-\sqrt{0.5}$
δ	$2/9$	$2/9 - \pi/4$
μ	3^{-1}	3^{-12}

To see why these are "natural," some theory is needed.

From Feynman's Nobel speech: "If every individual student follows the same current fashion in expressing and thinking about electrodynamics or field theory, then the variety of hypotheses being generated to understand strong interactions, say, is limited. Perhaps rightly so, for possibly the chance is high that the truth lies in the fashionable direction. But, on the off-chance that it is in another direction - a **direction obvious from an unfashionable view of field theory** - who will find it? Only someone who has sacrificed himself by teaching himself quantum electrodynamics from a peculiar and unusual point of view; one that he may have to invent for himself."

From the same speech:

“I remember that when someone had started to teach me about **creation and annihilation operators**, that this operator creates an electron, I said, "how do you create an electron? It **disagrees with the conservation of charge"**, and in that way, I blocked my mind from learning a very practical scheme of calculation.”

We will use this as the unfashionable idea.

Higgs model gives masses from arbitrary vacuum expectation values.

String theory also has too many vacua, 10^{500}.

But Koide relation shows **vevs are not arbitrary**.

So follow Feynman instinct and get rid of creation and annihilation operators and vacuum.

Preons eliminate need for Higgs:

"This problem **does not exist in preon models** for quark and lepton substructure with composite Z_0 and Ws, which, consequently, also avoid all other theoretical complications and paradoxes with the Higgs mechanism."

"Higgs pain? Take a preon!" hep-ph/9709227

J.-J. Dugne, S. Fredriksson, J. Hansson, E. Predazzi

The square root in the mass formula “suggests that the charged lepton mass spectrum is not originated in the Yukawa coupling structure at the tree level, but it is **given by a bilinear form**” Y. Koide, hep-ph/0506247

Density matrices are bilinear.

Spinors and density matrices have complementary linearity. Simple linear problems in one can be nonlinear in the other.

Density operators allow unification of mathematics (in a Clifford algebra). Allows unification of physics concepts.

	Spinor	Density
Pure State:	Vector	Operator
Mixed State:	(can't)	Operator
Operator:	Operator	Operator
Potential	Potential	Operator

Getting density from spinor is easy:

$$\rho_A = |A\rangle\langle A|$$

To get spinor from density, follow J. Schwinger, *Quantum Kinematics and Dynamics*, choose arbitrary constant "vacuum" pure state (primitive idempotent) $(\rho_0)^2 = \rho_0$ and define:

$$\begin{aligned} |A\rangle &= \rho_A\,\rho_0, \\ \langle A| &= \rho_0\,\rho_A \end{aligned}$$

for any pure state ρ_A.

Example. Choose arbitrary vacuum as projection for spin in $+z$:

$$\rho_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ket $(a,b)^\dagger$ becomes the density operator ρ_{ab}:

$$\rho_{ab} = \begin{pmatrix} a^* & b^* \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a^*a & b^*a \\ a^*b & b^*b \end{pmatrix},$$

Converting back to spinor form we have

$$|ab\rangle = \rho_{ab}\rho_0 \quad = \quad \begin{pmatrix} a^*a & 0 \\ a^*b & 0 \end{pmatrix} = a^* \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix},$$

$$\langle ab| = \rho_0\rho_{ab} \quad = \quad \begin{pmatrix} a^*a & b^*a \\ 0 & 0 \end{pmatrix} = a \begin{pmatrix} a^* & b^* \\ 0 & 0 \end{pmatrix}$$

See *Clifford Algebras and Spinors* by Pertti Lounesto. The factors of a and a^* are just a rescaling, we leave them off.

An amplitude $\langle ab|M|cd\rangle$, in density formalism becomes:

$$\begin{aligned} \rho_0\rho_{ab}M\rho_{cd}\rho_0 &= \begin{pmatrix} a^* & b^* \\ 0 & 0 \end{pmatrix}\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}, \\ &= \langle ab|M|cd\rangle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \langle ab|M|cd\rangle\ \rho_0 \end{aligned}$$

In density formalism, amplitudes are operators. Complex numbers are no longer just complex numbers, they are operators, just like the states (a unification).

Under rotation by 2π, spinors negate themselves:

$$
\begin{aligned}
|A\rangle &\rightarrow -|A\rangle, \\
\langle A| &\rightarrow -\langle A|, \\
\text{however} & \\
|A\rangle\langle A| &\rightarrow +|A\rangle\langle A| = \rho_A, \\
|0\rangle\langle 0| &\rightarrow +|0\rangle\langle 0| = \rho_0, \\
|A\rangle \equiv \rho_A\rho_0 &\rightarrow \rho_A\rho_0.
\end{aligned}
$$

In density theory, spinors change sign because spinor theory forgets to rotate the vacuum and $\langle A|0\rangle$ changes sign. Naughty spinor behavior is a normalization issue, not a part of reality.

$$\sigma_x\sigma_y\sigma_z = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix},$$

so we can eliminate complex numbers from Pauli algebra by replacing i with $\sigma_x\sigma_y\sigma_z$.

Get rid of the arbitrary choice of representation completely and write everything as sums over scalar (real) multiples of products of σ_x, σ_y, and σ_z.

No arbitrariness in representation of operators. Everything written in geometric terms now (i.e. scalar, vector, pseudovector, pseudoscalar). No spinors. Same can be done to Dirac algebra.

Find eigenstate of σ_y with eigenvalue +1:

$$\sigma_y \,(1+\sigma_y) \;=\; (\sigma_y + \sigma_y^2) \;=\; +(1+\sigma_y).$$

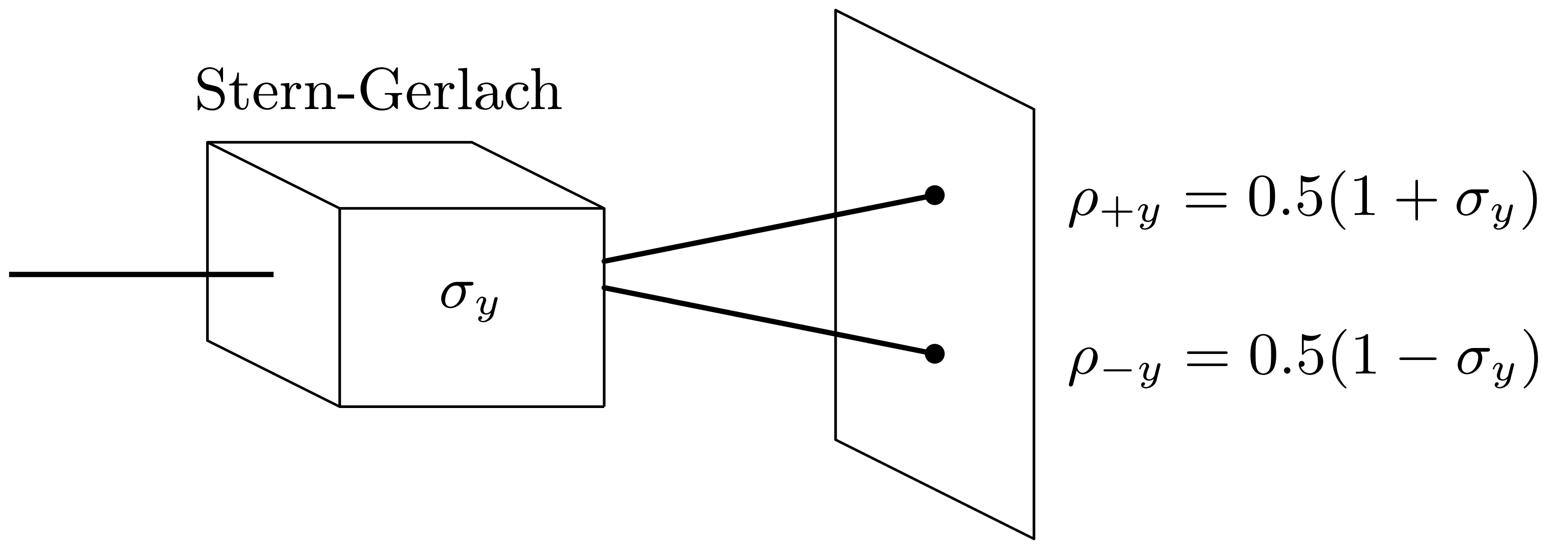
Therefore eigenstate must be multiple of $(1+\sigma_y)$. We require $\rho^2 = \rho$ so compute:

$$(1+\sigma_y)^2 \;=\; 1 + 2\sigma_y + \sigma_y^2 \;=\; 2(1+\sigma_y)$$

therefore,

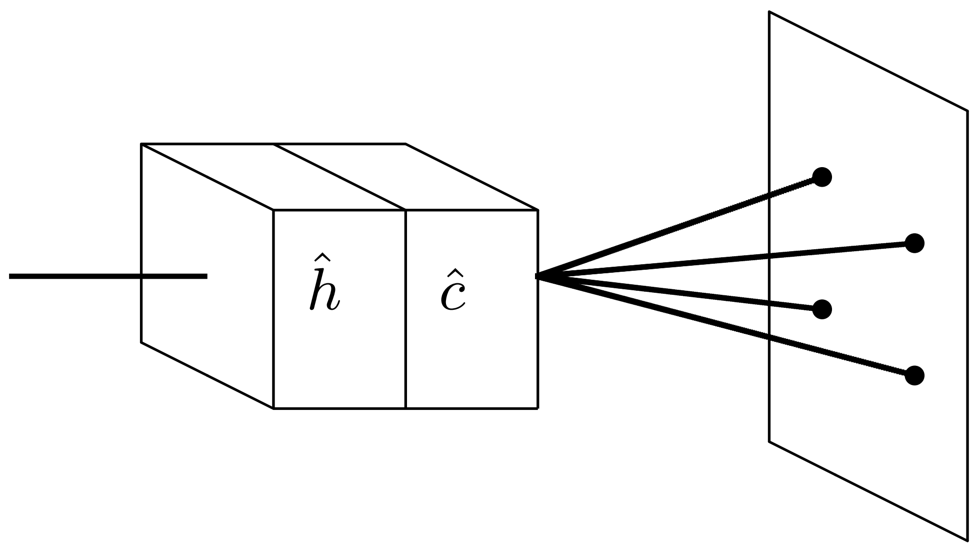
$$\rho_{+y} \;=\; 0.5(1+\sigma_y).$$

Stern-Gerlach beam splitter:



$0.5(1+\sigma_y)$ defines the state of a particle. It is an operator that projects out a portion of the beam. And it somehow describes the Stern-Gerlach apparatus itself (a field configuration).

Chiral Dirac beam splitter, splits to eigenstates of helicity $\hat{h}$, and charge $\hat{c}$:



$$e_R = .25(1+\hat{h})(1+\hat{c})$$
$$e_L = .25(1-\hat{h})(1+\hat{c})$$
$$\bar{e}_L = .25(1+\hat{h})(1-\hat{c})$$
$$\bar{e}_R = .25(1-\hat{h})(1-\hat{c})$$

Splitting pattern is square. To get cube, need three commuting operators. 8×8 representation instead of 4×4.

Since elementary particles are point particles, preons inside one share the same position. Need to define a potential energy that works between particles that share same position.

Two particles with opposite charges, for example $0.5(1+\hat{c})$ and $0.5(1-\hat{c})$ must attract. But these idempotents annihilate. So a first guess is to define potential energy as the transition probability:

$$V_1(A,B) = |\langle A|B\rangle|^2.$$

A second guess is to think of the Stern-Gerlach experiment. The sum of $0.5(1+\hat{c})$ and $0.5(1-\hat{c})$ is 1, so subtract 1 from the sum of the two operators and compute the matrix square:

$$\begin{aligned} V_2(A,B) &= |a_{11}+b_{11}-1|^2 + |a_{12}+b_{12}|^2 \\ &\quad +|a_{21}+B_{21}|^2 + |a_{22}+b_{22}-1|^2. \end{aligned}$$

A third guess is to modify V_2 by writing it in geometric form, ignoring scalar part:

$$V_3(A,B) = |a_x + b_x|^2 + |a_y + b_y|^2 + |a_z + b_z|^2.$$

For normalized states, these three guesses are equivalent:

$$V_1(A,B) = V_2(A,B)/2 = V_3(A,B).$$

Having the scalar weighted differently from the non scalar elements suggests we should allow the various blades to have different weights. The scalar gets much the smallest weighting. It will be the gravitational mass.

The leptons are made from collections of primitive idempotents that sum to give totals that are purely scalar. One can show that this rule gives the correct structure for the elementary particles, but it is easier to just show that the elementary particles have primitive idempotents that sum to scalars. These scalars, when squared, give the masses.

Let $\psi(z,t)$ be a plane wave moving at speed 1 in the $+z$ direction:

$$\psi(x,y,z,t) = \psi(z-t).$$

If ψ solves massless Dirac equation:

$$\begin{aligned} 0 &= (\gamma^0\partial_t + \gamma^1\partial_x + \gamma^2\partial_y + \gamma^3\partial_z)\psi(z-t), \\ &= (-\gamma^0 + \gamma^3)\dot{\psi}(z-t), \end{aligned}$$

then the general solution (with $-+++$ signature) is:

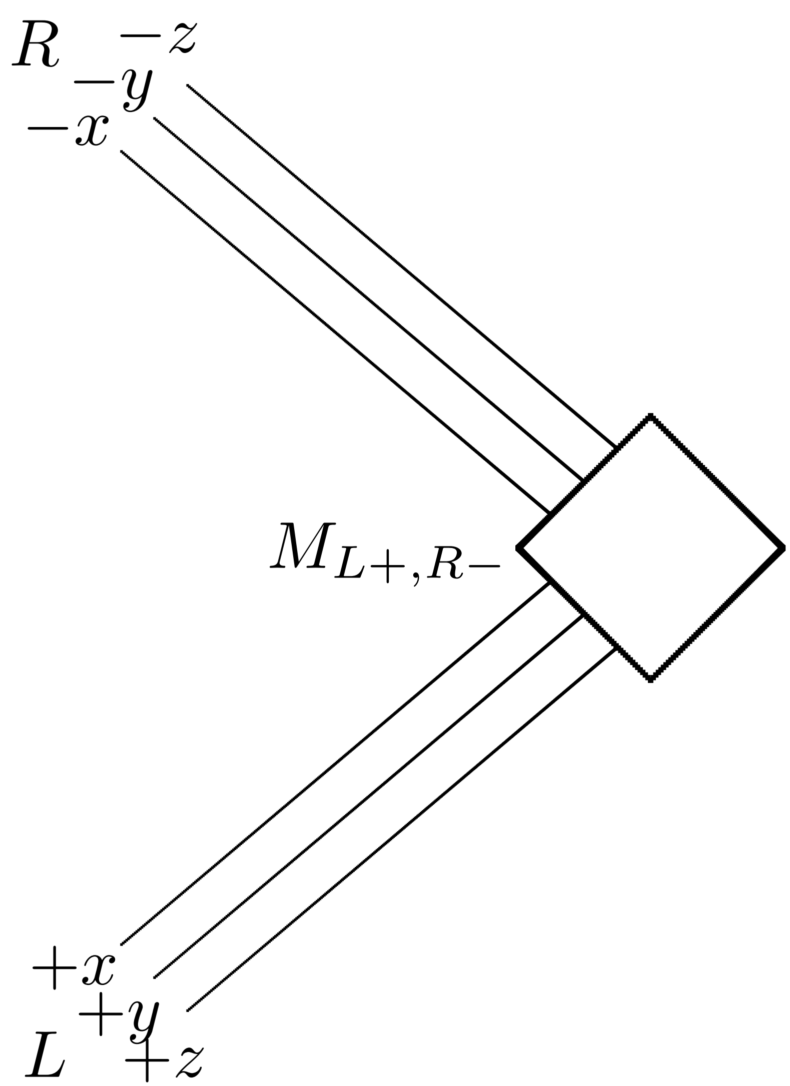
$$\dot{\psi}(z-t) = 0.5(1+\gamma^3\gamma^0)f(z-t),$$

Therefore, velocity is a geometric attribute of Dirac algebra. This was the basis of the old "zitterbewegung" models of the electron. And $0.5(1+\gamma^3\gamma^0)$ is a projection operator.

The Pauli exclusion principle for electrons allows only 2 states to coexist. Classically, we expect 6 non interfering waves to coexist, going in the $\pm x$, $\pm y$, and $\pm z$ directions.

Assigning three preons to the electron fills up the missing wave states.

Each of these preons can move back and forth by a sort of combined zitterbewegung movement. Also see Feynman's "checkerboard" model.



Left handed electron converts to right handed.

Three preons, in eigenstates of velocity in $+x$, $+y$, and $+z$ directions convert to three preons moving in $-x$, $-y$, and $-z$ directions.

Overall, electron is stationary.

The projection operators for velocity in the six directions are:

$$\begin{aligned}
\rho_{+x} &= 0.5(1+\gamma^1\gamma^0), \\
\rho_{+y} &= 0.5(1+\gamma^2\gamma^0), \\
\rho_{+z} &= 0.5(1+\gamma^3\gamma^0), \\
\rho_{-x} &= 0.5(1-\gamma^1\gamma^0), \\
\rho_{-y} &= 0.5(1-\gamma^2\gamma^0), \\
\rho_{-z} &= 0.5(1-\gamma^3\gamma^0),
\end{aligned}$$

For example, to transition from the $+x$ state to $-z$, the amplitude is the product: $\rho_{-z}\ \rho_{+x}$.

The matrix $M_{L+,R-}$ is just a matrix of transition amplitudes:

$$\begin{pmatrix} 0 & \rho_{-x}\rho_{+y} & \rho_{-x}\rho_{+z} \\ \rho_{-y}\rho_{+x} & 0 & \rho_{-y}\rho_{+z} \\ \rho_{-z}\rho_{+x} & \rho_{-z}\rho_{+y} & 0 \end{pmatrix}$$

Since $(1+\gamma^j\gamma^0)(1-\gamma^j\gamma^0) = 0$, the diagonal elements are all zero. That is, $0.5(1+\cos(\pi)) = 0$.

A similar matrix $M_{R-,L+}$ applies for getting from $R-$ back to $L+$, but $+s$ and $-$s are swapped.

To put this process into density form, we have to begin and end with the same state, for example $L+$. This is just the product, $M_{L+,L+} = M_{L+,R-}\ M_{R-,L+}$. Abbreviate $\rho_{+\chi}$ by χ, and $\rho_{-\chi}$ by $\bar{\chi}$:

$$M_{L+,L+} = \begin{pmatrix} x\bar{y}x + x\bar{z}x & x\bar{z}y & x\bar{y}z \\ y\bar{x}x & y\bar{z}y + y\bar{x}y & y\bar{x}z \\ z\bar{y}x & z\bar{x}y & z\bar{x}z + z\bar{y}z \end{pmatrix}.$$

For example, $x\bar{z}y = \rho_{+x}\rho_{-z}\rho_{+y} = 0.5(1 + \gamma^1\gamma^0)$ $0.5(1 - \gamma^3\gamma^0)\ 0.5(1 + \gamma^2\gamma^0)$.

Note that $\gamma^1\gamma^2\gamma^3\gamma^0$ squares to -1 and commutes with $\gamma^k\gamma^0$. So set $\hat{i} = \gamma^1\gamma^2\gamma^3\gamma^0$, a geometric imaginary unit. Reducing, we find:

$$M_{L+,L+} = \begin{pmatrix} \rho_{+x} & \eta e^{+\hat{i}\epsilon}\rho_{+x}\rho_{+y} & \eta e^{-\hat{i}\epsilon}\rho_{+x}\rho_{+z} \\ \eta e^{-\hat{i}\epsilon}\rho_{+y}\rho_{+x} & \rho_{+y} & \eta e^{+\hat{i}\epsilon}\rho_{+y}\rho_{+z} \\ \eta e^{+\hat{i}\epsilon}\rho_{+z}\rho_{+x} & \eta e^{-\hat{i}\epsilon}\rho_{+z}\rho_{+y} & \rho_{+z} \end{pmatrix}$$

where $\eta = \sqrt{0.5}$ and $\epsilon = \pi/4$, somewhat similar to the mass matrix. This is how I guessed the eigenvector form of Koide's relation. But the derivation is not faithful to its principles.

The operator acts on vectors of the form:

$$|a, b, c\rangle = (a\rho_{+x}, b\rho_{+y}, c\rho_{+z})^t,$$

where a, b, and c are complex numbers (using imaginary unit $\hat{i}$). The corresponding density operator form can be assumed to be circulant, one obtains:

$$|a, b, c\rangle \ \rightarrow \rho = \begin{pmatrix} a\rho_{+x} & b\rho_{+x}\rho_{+y} & c\rho_{+x}\rho_{+z} \\ c\rho_{+x}\rho_{+y} & a\rho_{+y} & b\rho_{+y}\rho_{+z} \\ b\rho_{+x}\rho_{+z} & c\rho_{+y}\rho_{+z} & a\rho_{+z} \end{pmatrix}$$

What we need to do is to solve $\rho^2 = \rho$. This might be different from complex matrices because of non commutativity.

After some algebra, one finds that, besides $\hat{1}$ and 0, there are six solutions. The primitive ones:

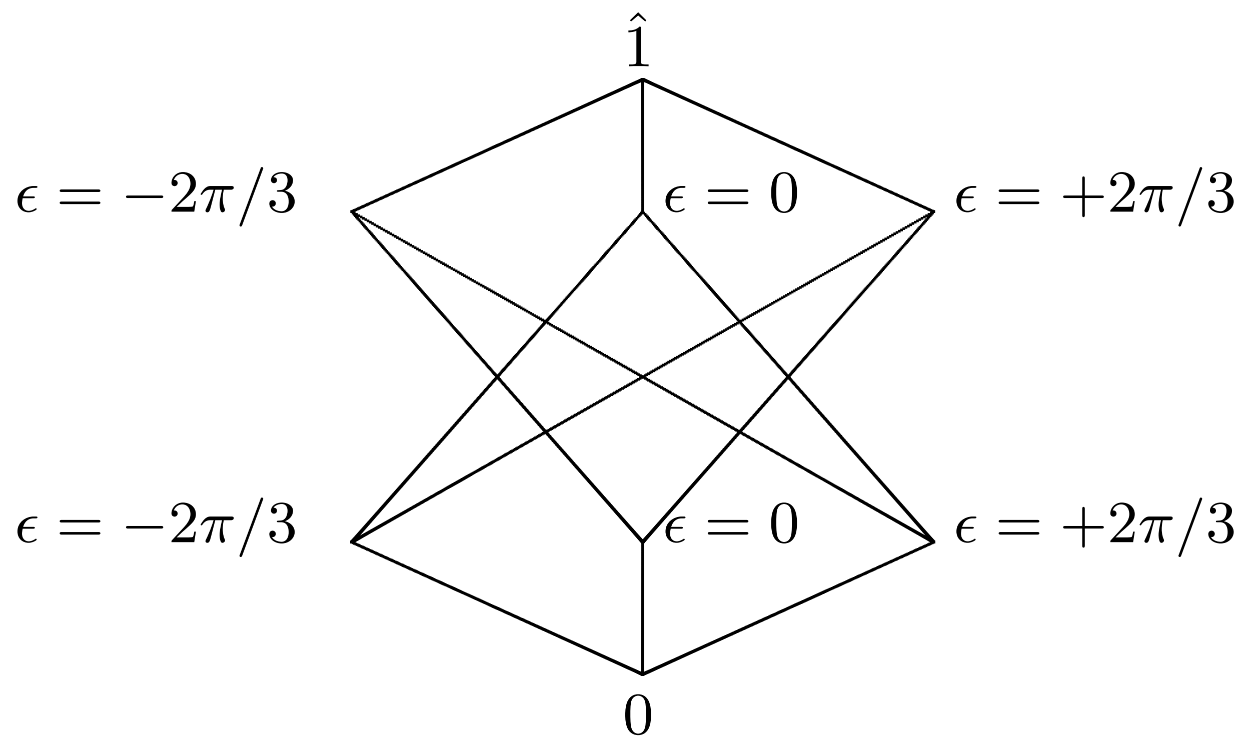
$$\frac{1}{3}\begin{pmatrix} \rho_x & \sqrt{2}e^{+\hat{i}\epsilon_n}\rho_x\rho_y & \sqrt{2}e^{-\hat{i}\epsilon_n}\rho_x\rho_z \\ \sqrt{2}e^{-\hat{i}\epsilon_n}\rho_y\rho_x & \rho_y & \sqrt{2}e^{+\hat{i}\epsilon_n}\rho_y\rho_z \\ \sqrt{2}e^{+\hat{i}\epsilon_n}\rho_z\rho_x & \sqrt{2}e^{-\hat{i}\epsilon_n}\rho_z\rho_y & \rho_z \end{pmatrix}$$

and the non primitive ones:

$$\frac{1}{3}\begin{pmatrix} 2\rho_x & -\sqrt{2}e^{+\hat{i}\epsilon_n}\rho_x\rho_y & -\sqrt{2}e^{-\hat{i}\epsilon_n}\rho_x\rho_z \\ -\sqrt{2}e^{-\hat{i}\epsilon_n}\rho_y\rho_x & 2\rho_y & -\sqrt{2}e^{+\hat{i}\epsilon_n}\rho_y\rho_z \\ -\sqrt{2}e^{+\hat{i}\epsilon_n}\rho_z\rho_x & -\sqrt{2}e^{-\hat{i}\epsilon_n}\rho_z\rho_y & 2\rho_z \end{pmatrix}$$

with $\epsilon_n = 2n\pi/3 + \pi/4$.

An aside: The eight solutions form the "lattice of propositions" for the quantum states:

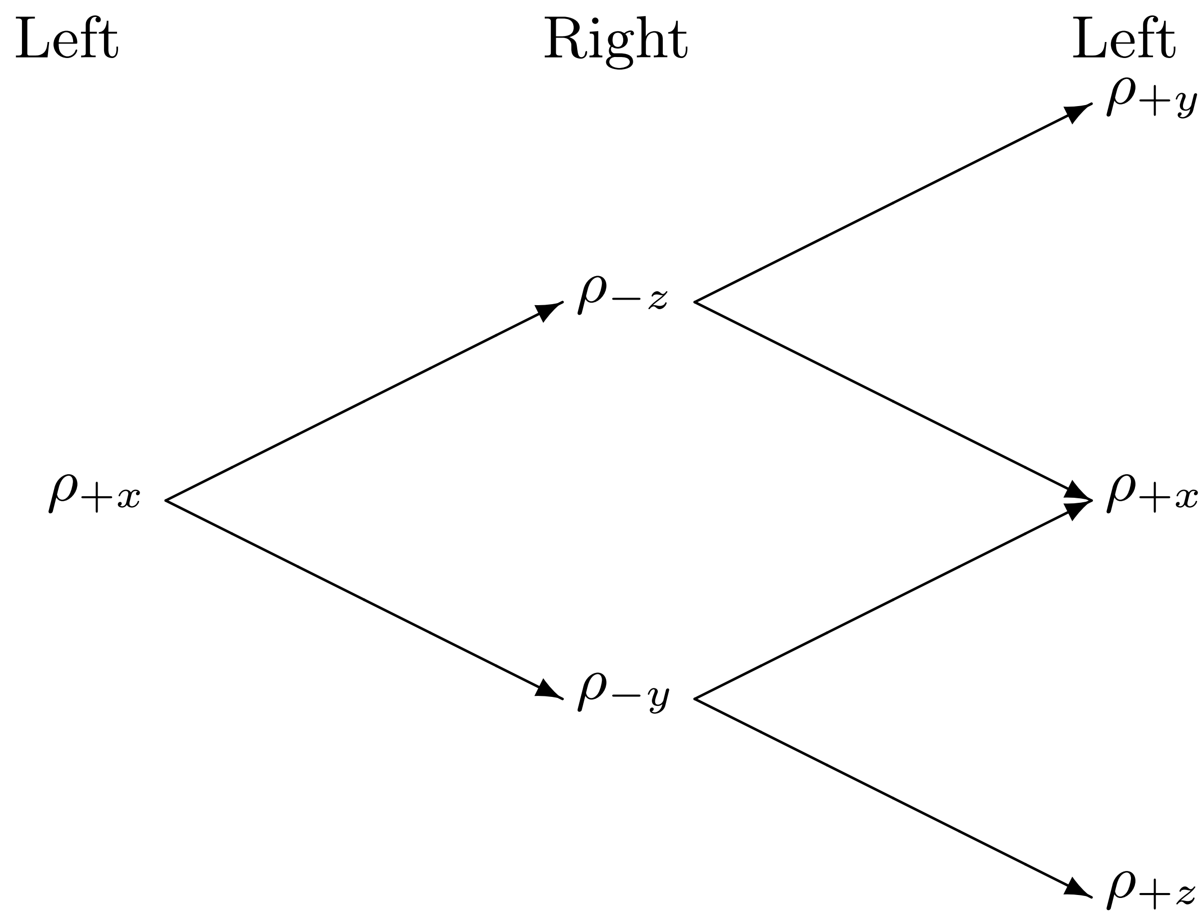


To calculate the potential energy, add up the nine components to get the square root of the potential energy:

$$\begin{aligned} V(n) &= 0.5(1+\sqrt{2}\cos(\epsilon_n)) && \text{scalar} \\ &+ \sqrt{2}(\gamma^1\gamma^0+\gamma^2\gamma^0+\gamma^3\gamma^0) \\ & (1+\sqrt{8}\cos(\epsilon_n)+\sqrt{2}\hat{i}\sin(\epsilon_n))/6. && \text{vector} \end{aligned}$$

The scalar part looks a like the Koide formula. The vector part would make the state have energy around the Planck mass. To get the mass of a stationary electron we have to add in the potential function for the right handed electron. Presumably this will cancel the vector part leaving the Koide-like scalar part.

These matrices were found under the assumption that the preons move around like this:



This is not quite compatible with what we know. The helicity operator $i\gamma_1\gamma_2\gamma_3\gamma_0$, the velocity operator $\gamma_3\gamma_0$ and the spin operator $i\gamma_1\gamma_2$ form a set of "commuting roots of unity". Along with $\hat{1}$, they form an Abelian group under multiplication. (They commute, so can be diagonalized.) We want the mass process to complement helicity and preserve spin; therefore it must complement velocity:

Left		Right		Left
ρ_{+x}	$\longrightarrow$	ρ_{-x}	$\longrightarrow$	ρ_{+x}

But this wouldn't allow the three preons to mix.

To preserve spin, but complement velocity and helicity, we look for an operator that commutes with spin, but anticommutes with velocity and helicity. There is only one solution, γ^0.

Since γ^0 anticommutes with the spatial vectors γ^1, γ^2 and γ^3 but commutes with γ^0, it generates the parity transformation.

Mass eigenstates have to be parity eigenstates. To get there from velocity eigenstates, we have to rotate the states. This is like getting $|+x\rangle$ from linear superposition of $|+z\rangle$ and $|-z\rangle$.

Rotating the velocity eigenstates produces two mass eigenstates. Maybe one transforms as:

$$L_1 \rightarrow R_1 \rightarrow L_1,$$

and the other transforms as:

$$L_0 \rightarrow R_0 \rightarrow \kappa\, L_0,$$

where κ is a 12th root of unity that explores 12 different degrees of freedom.

Density operator transformations are easy. No need to treat states different from operators. Let a be a real number, let χ be anything. Transform by:

$$M \rightarrow M' = e^{-a\chi} M e^{+a\chi},$$

The above transform preserves addition and multiplication, $\hat{1}$ and 0, and so maps primitive idempotents to primitive idempotents. Putting $\chi = \gamma^1\gamma^2$ gives rotations around z axis. Putting $\chi = \gamma^3\gamma^0$ gives boosts in z direction.

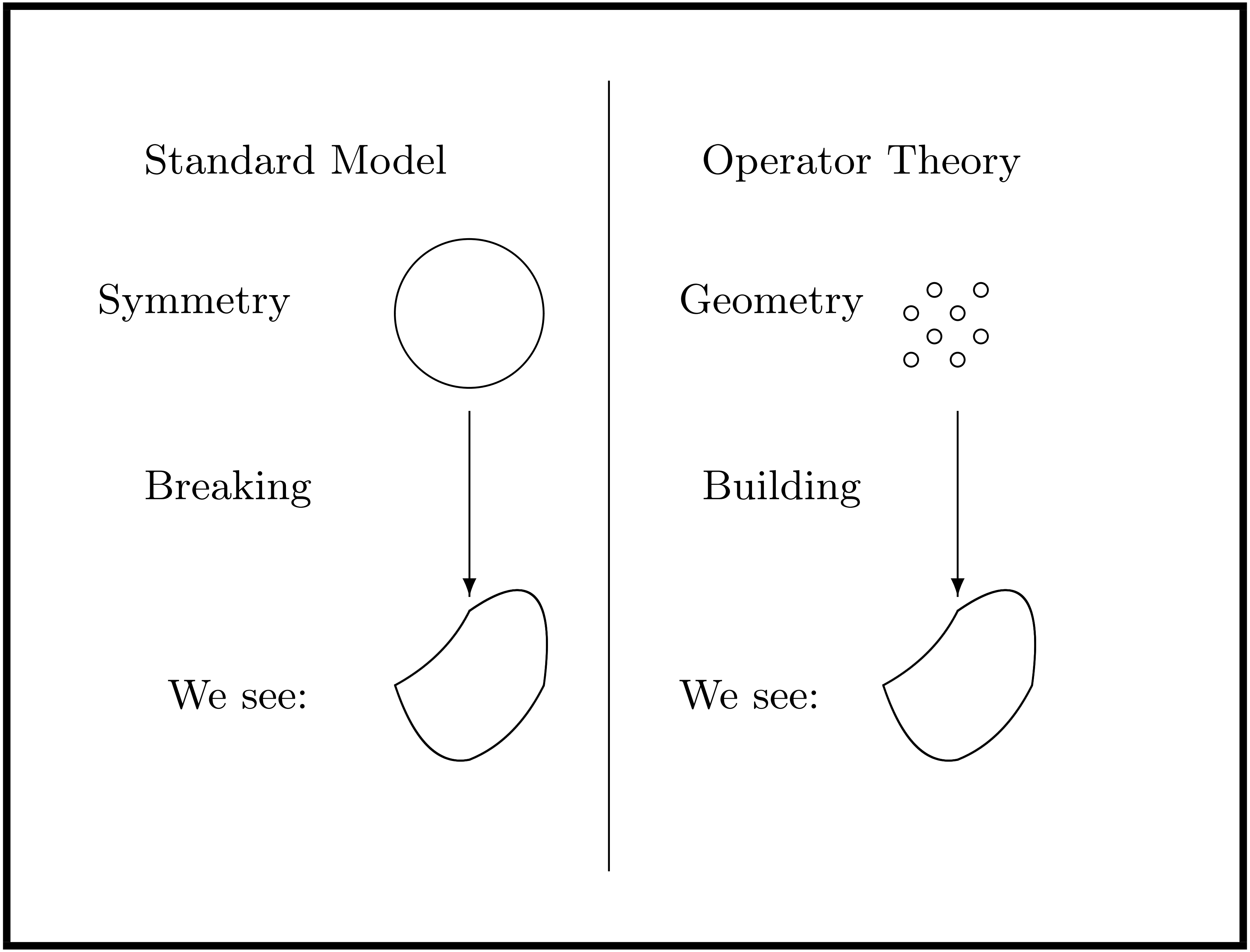
In Clifford algebra, we can make the (internal state) parity transformation into a continuous transformation:

$$\begin{aligned} M &\rightarrow e^{-a\gamma^0} M e^{+a\gamma^0}, \\ &= (\cos(a) - \sin(a)\gamma^0) M (\cos(a) + \sin(a)\gamma^0). \end{aligned}$$

When $a = \pi/2$ this is the usual parity transformation (using signature of $-+++$). This might be needed to give space time a preferred handedness.

Also, the quantum numbers for weak isospin won't work unless the velocity eigenstates need to be composites made up of two (different) primitive idempotents. Computer simulation of bound states where the potential has been modified by an exponential transformation show an interesting structure that can have this form.

That's about where things stand now. Incomplete but hopeful.


Standard Model
Operator Theory
Symmetry
Geometry
Breaking
Building
We see:
We see:

The ν_R is not shown for clarity.