The Proper Time Geometry

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The "Proper Time Geometry" (PTG) is described as an alternative to the usual "Lorentz-Minkowski Geometry" (LMG) of space-time. The PTG provides identical results in classical mechanical calculations, but promises simplified quantum mechanical structures due to the fact that all particle speeds are made identical to c. Examples of calculations in classical relativity are included.

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I. CAUSALITY AND LORENTZIAN RELATIVITY

Historically, quantum mechanics was developed on the same geometry of space-time that Einstein described in his 1904 paper on Special Relativity (SR), the Lorentz-Minkowski Geometry (LMG). Since then, the unification of gravitation and quantum mechanics has resisted all efforts. Perhaps the problem is with the use of the LMG. While this familiar geometry is conceptually the simplest that supports SR, it is not unique in that achievement. This paper describes an alternative geometry that does the same through the use of a hidden dimension.

The theory of Quantum mechanics gives a few clues that the LMG is not the correct one for describing a local geometry of space-time. The most obvious clue is that of causality. "Quantum field theory solves the causality problem in a miraculous way, ... We will find that, in the multiparticle field theory, the propagation of a particle across a spacelike interval is indistinguishable from the propagation of an antiparticle in the opposite direction. When we ask whether an observation made at point x_0 can affect an observation made at point x, we will find that the amplitudes for particle and antiparticle propagation exactly cancel – so causality is preserved." [1, page 14] Thus causality is preserved for the results of the computations of the theory, but they are not present in the mechanism itself.

The problem of causality arises due to the lack, in SR of a preferred reference frame. We will assume that there is a preferred reference frame, and therefore that the propagation of particles across spacelike intervals is not a causality problem. Of course we will also assume that the structure of space-time makes it impossible, or exceedingly difficult, to locate that preferred reference frame. From a calculational point of view, this small detail is no change from the current theory. It is a philosophical choice only, but a choice that will allow us to explore geometries other than the LMG. Nor is this choice of interpretation new, it dates to the dawn of relativity and is known as Lorentzian Relativity (LR). Lorentzian Relativity has been generalized to match General Relativity, but for the purposes of this paper, we will stick with a locally flat geometry.

Lorentzian relativity does not deny that the laws of physics appear identical in all inertial reference frames, but nevertheless asserts that there does exist a preferred reference frame. As with standard relativity, speed slows down the perceived passage of time, so that an object at rest in the preferred reference frame ages faster than objects moving with respect to it. The difference between Lorentzian relativity and standard relativity is essentially that standard relativity holds that time itself is slowed down by speed, while Lorentzian relativity holds that only the perception of time is slowed down by speed.

As a theory identical to standard relativity, Lorentzian relativity suffers from two defects. The first is that since it is presumed impossible to detect the preferred reference frame, there is no obvious reason to include it in a theory. The second is that the theory gave no explanation for why moving clocks would be slowed down. Against these defects, the primary advantages of the Lorentzian theory are that it eliminates the problems with causality between space-like separated events, and that it keeps space and time separate rather than mixing them into space-time. From a phenomenological point of view the two theories are identical, and since standard relativity has fewer assumptions, it is the preferred theory. But from an ontological point of view, it is difficult to deny that Lorentzian relativity is the better, especially with quantum mechanics indicating that particle influences, though not causal signalling, can exceed the speed of light.

II. THE PROPER TIME GEOMETRY

Both versions of special relativity share the same metric: [6]

$$ds^2 = dt^2 - (dx^2 + dy^2 + dz^2). \tag{1}$$

But the interpretation of the t coordinate is different between the two theories. With standard relativity, t is the time coordinate appropriate for this reference frame only. In Lorentzian relativity, assuming that the above reference frame is the preferred one, t is promoted to be the universal time for all reference frames. In both theories, the dimensional coordinates do not include s,

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which is instead defined as the metric distance. That s is not included as a dimensional coordinate is surprising, given that ds is the only measurement that observers on different reference frames can agree on.

It is therefore natural to promote s from a parameter that is specific to a particular moving body, to a coordinate that is generic to space-time. The time experienced, or age, of a moving body is therefore equivalent to the progress made in the s dimension. In order to allow for collisions between objects of different ages, the s dimension must be cyclic and small. Call the radius of the s dimension R_s. Since this topology results from promoting the proper time parameter s to a coordinate, it will be called the "Proper Time Geometry" (PTG) in this paper, and the relativity theory converted to it will be called "Proper Time Relativity" (PTR).

These three versions of relativity share the same metric, Eq. (1), but with a different interpretation of the variables. While SR treats t as a coordinate, both Lorentzian relativity and PTR promote t to a special status, universal time. Where LR and PTR differ is in the treatment of s. Since the PTG treats s as a coordinate, it is natural to group it with the other coordinates and rewrite the Minkowski metric in this form:

$$dt^2 = ds^2 + dx^2 + dy^2 + dz^2. \qquad (2)$$

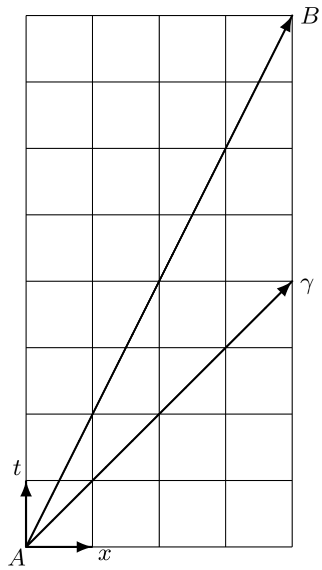
The above equation gives the amount of time that is needed for the movement of a massive or massless body by (ds, dx, dy, dz). In the PTG therefore, all bodies move at the speed of light, but part of that movement is in the hidden dimension. Various odd attributes of the theory of relativity, such as the huge amount of energy present in matter, and the impossiblity of matter exceeding the speed of light, become natural consequences of a universal speed for all matter and energy. And since the equation of motion of waves are much simpler when the waves are presumed to all travel at the same speed, the wave mechanics version of the PTG promises to be simpler than wave mechanics in the LMG.

III. WICK ROTATIONS

Another geometrical clue from quantum mechanics is the use of the Wick rotation. A Wick rotation consists of promoting time t from a real variable to a complex one, and then calculating with imaginary time. It is miraculous that this mathematical artifice simplifies, rather than complicates quantum mechanics. Wick rotations are common enough to be found in almost any modern QFT book, [1, pp 292-3] [2, pp 475-6] [3, pp 12,261] and the technique is universal in lattice gauge theory, [4] the only technique at present for calculating the properties of composite particles such as the mass of the proton.

A Wick rotation changes the Lorentz-Minkowski 4-dimensional geometry into a 4-dimensional Euclidian geometry. With this change, a quantum mechanical theory becomes a statistical mechanical theory with $\hbar$ playing the role of the temperature. The only difference between the Wick rotated LMG and the PTG is that the PTG makes the 4th dimension cyclic, while a Wick rotation keeps it unbounded. In actual practice with Lattice Gauge theory, even this difference is eliminated as their simulations are done with all four dimensions cyclic. The implication is that $\hbar$ should be interpreted as the temperature of background fluctuations in space-time.

FIG. 1: Particle moves from A to B in Lorentz-Minkowski Geometry. The world line of a photon is shown as γ. Horizontal coordinate is x, a spatial position. Vertical coordinate is t, the time coordinate of this particular reference frame. Proper time is a derived value, not a part of the topology, and so is not shown.

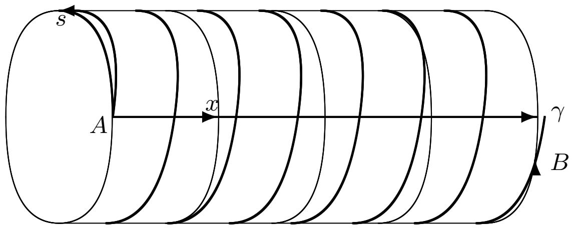


IV. PARTICLE PATHS

The geometrical difference between SR and PTG can be illustrated with figures showing how the two theories describe the motion of a particle in one dimension. With SR, the relevant coordinates are x and t, while s is only a calculated parameter and does not appear in the geometry of space-time. Positions in space-time include t, and are described as events. For a body moving at half the speed of light, from event A to event B a familiar illustration showing the world line of the body, along with the world line of a photon simultaneously emitted at A in the same direction is illustrated in Fig. 1. The metric length of the photon path is zero, while the path of the particle moving from A to B is $\sqrt{(8^2 - 4^2)} = 6.9$, which is the amount of proper time experienced by the particle.

The same situation for the Proper Time Geometry is shown in Fig.2. Proper time s has been promoted to a coordinate, giving the particle an extra degree of freedom that is cancelled by the added requirement that the particle move at the speed of light. Since the particle re-

FIG. 2: Particle moves from A to B in Proper Time topology. The path of a photon is shown as γ. Horizontal coordinate is x, a spatial position. The circular coordinate is s. Global time is a parameter, not a part of the topology, and so is not shown. The proper time experienced by the particle can be found by counting the number of trips it makes around the s dimension, 6.9 in this illustration.



quires 8 units of time to traverse the path from A to B, the length of that helical path is 8 units long. The particle experiences a passage of time given by the distance it travels in the s direction. As with the LMG case, the proper time interval is about 6.9 units of time. The scale for the s dimension has been shown so that one revolution is one unit of time. Consequently, the path of the particle winds around the s dimension a little less than 7 revolutions. The photon does not experience the passage of proper time, so its path does not wind around the s dimension, and its path is coincident with the x axis. The length of the path traversed by the photon is 4 units, so the time required for its traversal is 4.

The Proper Time Geometry adds an extra variable to standard relativity, but the maximum size of that variable goes to zero as $R_s \to 0$. In later papers, when considering wave behavior and quantum mechanics, R_s will play the role of a parameter in the theory, but for the purpose of matching the calculations of SR, no particular value of R_s is needed, we require only that it be sufficiently small. Therefore, as far as matching LMG, this theory has no change in the number of free parameters.

When converting a classical mechanical calculation from the PTG to LMG, the value of the s coordinate is simply ignored. If R_s is infinitesimal, the error in doing this will also be infinitesimal. When converting to the PTG, the s value can be arbitrarily set to zero, again with an infinitesimal error. Since the errors in converting between the two geometries is infinitesimal, they cannot be distinguished by any classical measurement. Examples of calculations using the PTG are included in the Appendix.

V. NEW FOUNDATION FOR QUANTUM MECHANICS

With the PTG, the Lorentz symmetry of space-time is broken in three obvious ways. First, as with Lorentzian relativity, a preferred reference frame is assumed. This suggests thought experiments that could distinguish a preferred reference frame. Second, very short distances tend, on average, to be longer than LMG would give. Third, time, or at least the perceived passage of time for a moving body, may have a discrete nature to it. While losing Lorentz symmetry might be seen as a severe defect to a theory, each of these differences has been suggested before. The preferred reference frame, or ether, is popular in quantum theories. And recently the loop quantum gravity theories assume both a complete failure of the metric for short distances, and that time will advance in discrete steps. In comparison, the changes to the standard model of quantum mechanics described in this paper are quite limited. In addition, an infinitesimal violation of Lorentz symmetry has the effect of allowing a quantum theory based on the geometry to avoid the Coleman-Mandula "no go" theorem. [5] This suggests that an explanation for the internal symmetries of QM may exist in the external geometry of space-time as will be demonstrated in a following paper.

This paper has been entirely about space, time and movement, rather than momentum or energy. There are several reasons for this. The mass, energy and momentum of elementary particles are radically modified by the Quantum Field Theory process of renormalization or resummation. By contrast, space and time are not, so it is likely that as far as looking at a unifying field theory, we can expect our large scale idealization of space and time to be a more stable guide than our idealizations of mass, momentum and energy. The equations of SR having to do with matter can be translated directly over into the PTG, but that the result is not simplified we attribute to resummation effects as will be described in a later paper.

That the PTG can be used for classical mechanics gives hints about the underlying structure of particles in quantum mechanics. A Wick rotation converts Lorentz boosts into Euclidian rotations in 4 dimensions. Thus it is natural that in the PTG all particles travel at the same speed, c, and the underlying field theory should be one where all particles of the same type possess the same momentum. This would explain the mystery of why the standard model requires "chiral" wave states. The chiral wave states are massless and handed, and travel at the speed of light, and it is these wave states, that are created and annihilated at vertices. The choice of chiral wave states depends on the frame of reference, an ontological problem that theories with a preferred frame of reference avoid. The effect is that "relativity" applies not to physics and space-time, but instead only to the results of calculations.

Later papers will describe a new foundation for quantum mechanics based on the insights available from the PTG.

APPENDIX: CLASSICAL SPECIAL RELATIVITY CALCULATIONS IN PTG

In order to make clear how calculations in the PTG can match those of the LMG, this appendix provides detailed calculations for time dilation and length contraction using both techniques. Lorentz relativity caclulations are identical to those of SR.

1. Time Dilation

Problem: A spaceship travels 3 light years away form earth, at a speed of $0.6c$, and then returns at the same speed. What is the proper time experienced on the Earth during the voyage, and what is the proper time experienced on the spaceship?

Special Relativity Solution: The voyage requires $3/0.6 = 5$ years each way for a total of 10 years. This is the proper time experienced on the Earth. The spaceship experiences a time dilation of $(1 - 0.6^2)^{0.5} = 0.8$, so the proper time experienced on the spaceship is $10 \times 0.8 = 8$ years.

Proper Time Solution: The spaceship starts at the point $(x, y, z, s) = (0, 0, 0, 0)$. Align the x axis with the direction of travel. The velocity of the spaceship on the outgoing voyage is therefore given by the vector $(0.6, 0, 0, 0.8)$. The 0.8 value is required to make the speed of the spaceship work out in total be 1. The spaceship's position as a function of the global time t is therefore:

$$(0, 0, 0, 0) + (0.6, 0, 0, 0.8)t_1 \tag{A.1a}$$

Setting this equal to $(3, 0, 0, s_1)$ gives t_1, the global coordinate time for the arrival of the spaceship at its destination, and t_1 is therefore 5 years. Note that the value of s_1 is unspecified, as the total length of the hidden dimension is negligible as compared to the many light years of travel. Since the proper time component of the velocity of the spaceship is 0.8, the total elapsed proper time on the outgoing voyage of the spaceship is therefore $0.8 \times 5 = 4$ years. Similarly, the return trip uses a velocity of $(-0.6, 0, 0, 0.8)$ and results in a coordinate time passage of 5 years and a proper time for the spaceship of another 4 years. The result is, of course, identical to the Special Relativity result.

2. Lorentz Contraction

A rod flies lengthwise through a laboratory with a speed of $0.923c$. The lab measures the length of the rod as 6 meters. How long is the rod measured in a coordinate system moving with the rod?

Special Relativity Solution: The Lorentz contraction factor is $(1 - 0.923^2)^{-0.5} = 2.6$, so the proper length of the rod is $6m \times 2.6 = 15.6$ meters.

Since the Proper Time topology does have a preferred coordinate system, the question is not as clear as it is in special relativity. But in any given coordinate system, the constancy of the speed of light provides a technique for measuring length. Accordingly, the rod can be measured in its own frame of reference by calculating the time required for light to travel the length of the rod. Since proper time is a property of individual particles, rather than dimensional objects such as rods, the length of the rod will have to be measured by computing the time required for the light to travel down the rod, be reflected at the end, and then travel back to the point of origin on the rod. The proper time experienced by the end point of the rod during this flight will indicate (when multiplied by c) twice the length of the rod.

So let the rod begin at position $(0, 0, 0, 0)$ through $(6m, 0, 0, 0)$, and set the velocity vector for the rod to be $(0.923, 0, 0, 0.384)$ so that it moves in the $+x$ direction. The light signal starts at $(0, 0, 0, 0)$ and proceeds with a velocity vector of $(1, 0, 0, 0)$ until it meets with the other end of the bar at time t_1. The light direction is then reversed, and it travels with velocity $(-1, 0, 0, 0)$ until it meets up with the trailing end of the bar at time t_2. The length of the bar, in the reference frame of the bar, is then 1/2 the proper time experienced by the trailing end of the bar from 0 to t_2. The equations for t_1 and t_2 are therefore:

$$\begin{aligned} (0,0,0,0) &+ (1,0,0,0)t_1 \\ = (6,0,0,0) &+ (0.923,0,0,0.384)t_1, \end{aligned} \tag{A.2a}$$

$$\begin{aligned} (1,0,0,0)t_1 &+ (-1,0,0,0)(t_2 - t_1) \\ = (0,0,0,0) &+ (0.923,0,0,0.384)t_2. \end{aligned} \tag{A.2b}$$

Since our world does not distinguish between the hidden proper time coordinate, the equalities need only be established for the first three coordinates.

The solution is $t_1 = 78$ meters, and $t_2 = 81.12$ meters. The proper time experienced on the trailing edge of the rod is, by time dilation, 0.384 of t_2, which gives 31.2. Half of this is the proper length of the bar, which is the same as the value given by special relativity. Therefore, both theories show the Lorentz contraction of the bar to be the same. Since the two theories are the same in both time dilation and Lorentz contraction, any dynamical problem can be converted between them, and the results of standard relativity translate directly. Thus the problem can be worked with the simpler methods of SR with only a philosophical difference.

[1] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley Publishing Company, 1995).
[2] S. Weinberg, *The Quantum Theory of Fields, Volume I Foundations* (Cambridge University Press, 1995).
[3] A. Zee, *Quantum Field Theory In A Nutshell* (Princeton University Press, 2003).
[4] H. J. Rothe, *Lattice Gauge Theories: An Introduction* (World Scientific Publishing, 1997).
[5] Coleman and Mandula, Physical Review **159**, 1251 (1967).
[6] I've chosen the time-like version of the metric because this is the one that is natural for describing the paths of moving objects.