Phase Velocity of de Broglie Waves

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It's been long known that the phase velocity of de Broglie's matter waves exceeds the speed of light. Modern analysis ignores the phase velocity in favor of the group velocity, since this is what corresponds to particle speeds. This paper examines the problem from the point of view of a hidden dimension.

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This is the fourth paper in a series by the author describing a new foundation for quantum mechanics based on the Proper Time Geometry (PTG). However, this paper is independent and can be read without reference to the previous papers. For an introduction to the PTG and a brief discussion of how clasical relativistic mechanics works in the geometry, see [1]. For a geometric description of the internal symmetries of the fermions, see [2]. For an explanation of how the symmetries of charge conjugation and parity complementation come to be violated, see [3]. For work by other authors using similar modifications of the geometry of special relativity see [4], [5], [6], [7].

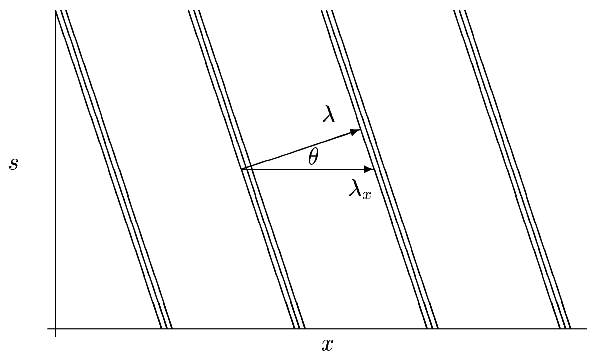
In 1923, L. de Broglie proposed that matter possesses the same wave particle duality as light, with similar relations between frequency and energy. [8] As a consequence of time dilation and the relation $E = \hbar\omega$, [10] de Broglie concluded that there must be associated with a particle travelling at speed v, a wave that travels at (phase) speed:

$$v_\phi = c/\sqrt{1-v^2} = c/\beta. \tag{1}$$

Since this speed is greater than c, he referred to the wave as fictitous. The conventional answer to this oddity is to note that the group velocity of a wave packet will be less than c, and to stress that this is what must be associated with the velocity of the particle. For example, in A. Messiah's excellent introduction to quantum mechanics, the details of the calculation for group velocity are given, but he fails to explicitly mention that the phase velocity exceeds c. [9, CH.II, §3]

The presence of a hidden dimension in the PTG (and similar geometries) suggests that the explanation for the faster than light phase velocity is that the wave is being considered only in 3-dimensions. That is, a wave travelling with a phase velocity of c in the PTG, when translated into a wave in the usual space, will give a phase velocity in excess of c. This effect is commonly seen at the beach, where the velocity of breakers along the shore exceeds the phase velocity of the incoming waves. See Fig. (1).

FIG. 1: Illustration of the increase in phase velocity when a hidden dimension is ignored. The true wavelength in 2 dimensions, λ, is increased to $\lambda_x = \lambda/\cos(\theta)$ when only the x dimension is considered. The phase velocity is $\lambda/2\pi\omega$.



Adding a small hidden dimension to the usual geometry of space-time allows matter waves to travel with speed c, but with a phase velocity exceeding c in the usual 3 dimensions. The PTG includes just such a hidden dimension. From the point of view of relativity, the hidden dimension corresponds to proper time. The PTG shares the same metric as Lorentz geometry, but with the proper time, s, promoted to use as a coordinate instead of just a parameter:

$$dt^2 = ds^2 + dx^2 + dy^2 + dz^2. \tag{2}$$

Thus the space defined over the coordinates (x, y, z, s) is a metric space with signature $(++++)$. The hidden dimension s is cyclic, with a radius of R_s. Thus waves will be required to be periodic with period $2\pi R_s$.

In the PTG, all classical particles travel at speed c [1], and so are naturally associated with waves that travel at this same speed. Accordingly, let $k = (k_x, k_y, k_z, k_s) = (\mathbf{k}, k_s)$ be a wave vector, and let ω be the wave frequency, with the wave given by:

$$\psi(x, y, z, s; t) = \Re(\exp(i(k \cdot r - \omega t + \phi_0))), \tag{3}$$

where $r = (x, y, z, s) = (\mathbf{r}, r_s)$ is a position, ϕ_0 is a phase, and the use of the imaginary unit is only for convenience in calculation. The requirement that the phase velocity of the wave, in 4 dimensions, be c gives

$$c\,|k| = \omega. \tag{4}$$

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The phase velocity in 3-dimensions is given by the distance of one wavelength divided by the time for one cycle:

$$v_{\phi 3} = \left(2\pi/\sqrt{k_x^2+k_y^2+k_z^2}\,\right)/(2\pi/\omega) = c/(1-(k_s/|k|)^2). \quad (5)$$

This is greater than c, but if k_s is small compared to k, it is very close to c, corresponding to a particle that travels at close to the speed of light, which gives these waves a natural interpretation as the chiral fermions.

Since the s dimension is cyclic, by continuity we must have that:

$$k_s 2\pi R_s = 2\pi n \;\Rightarrow\; k_s = n/R_s, \quad (6)$$

for some n an integer. In the PTG, the various fermion families correspond to the terms in a Fourier series obtained by integrating over s [2]. We therefore require that $k_s = 1/R_s$ for fermions in the electron family, $k_s = 2/R_s$ for fermions in the muon family, etc. Note that k_s can be either positive or negative. Thus the speeds of the chiral fermions are not quite c, and, under the assumption that there is a single frequency shared among them, their speeds decrease with increasing family number:

$$v_n = c(1-(k_s/|k|)^2)^{1/2} = c(1-(n/|k|R_s)^2)^{1/2}. \quad (7)$$

In order to allow these waves to propagate in 3 dimensions at approximately c, we require only that $|k|R_s >> n$.

It is interesting that when a plane wave is considered in a 3−dimensional space, the points in that space where the phase of the wave are equal to a given constant form a series of parallel, unconnected planar surfaces. The case where $n = 0$, which corresponds to the bosons of the PTG, is similar. But for the fermions, the points in the PTG, where the phase is equal to a constant, are connected. This gives an intuitive explanation for how it is possible, in the standard model, that particles represented by plane waves can have movement despite the absence of any position or frequency dependence in the probability or momentum densities.

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[10] Some notation, for example the use of ω in preference to ν, has been changed to match modern usage.