# Schwinger's Measurement Algebra, Clifford Algebra, Preons and the Lepton Masses

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A common complaint about the standard model is that it has too many parameters.

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Today I will be discussing the lepton masses. They depend on a Higgs vev and 6 Yukawa coupling constants, a total of 7 arbitrary parameters.

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This is a plot of the weak hypercharge and weak isospin numbers of the elementary fermions of the first generation. Kind of looks like a cube.

The quarks show up in the columns, and they are in between two leptons so it's natural ...

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... to suppose that the quarks and leptons are composite, with the leptons bound states of three similar preons each, and the quarks bound states of three distinct preons each.

This sort of preon model is a bit ugly when it comes to assigning spin to the preons and stuff like that. Maybe they're spin one sixth.

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Since the three preons making up a lepton are all similar, a natural operator type to use for cross generation is the circulant. If we want our operator Gamma to have real eigenvalues, then we can write it this way. Here, mu is an overall scale. If this were a table of coupling constants, eta would be an amplitude. Delta is a phase that shows up when you take two different paths to get somewhere.

The eigenvectors of a circulant matrix are here, where omega is the complex cubed root of one. The resulting eigenvalues are here.

The trace gives the sum of the eigenvalues, and we can square the matrix and take the new trace to get the sum of the squares of the eigenvalues.

A 3x3 circulant operator in complex numbers:

$$\Gamma(\mu,\eta,\delta) = \mu \begin{pmatrix} 1 & \eta e^{+i\delta} & e^{-i\delta} \\ e^{-i\delta} & 1 & \eta e^{+i\delta} \\ \eta e^{+i\delta} & e^{-i\delta} & 1 \end{pmatrix}.$$

The eigenvectors:

$$|n\rangle = \begin{pmatrix} 1 \\ e^{+2in\pi/3} \\ e^{-2in\pi/3} \end{pmatrix} \quad n = 1, 2, 3$$

The eigenvalues:

$$\lambda_n = \mu(1 + 2\eta\cos(\delta + 2n\pi/3))$$

The result is that we can cancel out mu and eliminate delta and get this relationship between the eigenvalues and eta:

Our eigenvalue formula had a cosine in it, so it could produce negative values. Now mass is always positive, so lets put the square roots of the masses of the charged leptons into our formula and see what value we get for eta squared. And then we can get mu and delta as well. I'm putting a one as a subscript because these are for the charged leptons, I'll use zero for the neutrinos.

As you can see, these numbers don't look at all random. The first, was found by Yoshio Koide in 1982. I found the 2/9 this past summer.

$$\eta^2 = \frac{3}{2}\frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{(\lambda_1 + \lambda_2 + \lambda_3)^2} - \frac{1}{2} = \eta^2.$$

$$\frac{3}{2}\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} - \frac{1}{2} = \eta_1^2 = 0.5000018$$

$$\delta_1 = 0.2222220$$

The way Koide found his relationship is in this form:

There are three unknown neutrino masses but we have two pieces of oscillation data. It's natural to use the Koide relation to predict the neutrino masses and several papers are out there having tried to do this. But they were using Koide's equation, not my eigenvector form, and it turns out that the oscillation data excludes this.

If you use the eigenvalue version of the mass relationship, it is natural to suppose that one of the square roots of mass is negative. Then you can fit the masses. A set of values that meet recent oscillator data, and gets the Koide relationship with one negative square root is here:

Translating back into the eigenvalue language, the neutrino mass values were chosen to get eta squared to be one half. It's really not very surprising that adding that extra freedom allowed the equation to be satisfied.

$$\frac{(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau})^2}{m_e+m_\mu+m_\tau}=\frac{3}{2}.$$

$$\begin{aligned} m_1 &= 0.0004\ \ \mathrm{eV}, \\ m_2 &= 0.009\ \ \mathrm{eV}, \\ m_3 &= 0.05\ \ \mathrm{eV}. \end{aligned}$$

$$\frac{(-\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3})^2}{m_1+m_2+m_3} = \frac{3}{2}.$$

We can take those predicted neutrino masses and use them to calculate mu and delta for the neutrino sector. The result is fairly close to:

Now these factors of three might have something to do with those braids that Lee Smolin was talking about yesterday. Maybe the electron sector has twelve braid stages while the neutrino sector has one, and this might have something to do with this angle of pi over twelve. Who knows. No one had an explanation for the Koide mass relation and no one has an explanation for this one.

We can suppose that these formulas are exact, and that eta squared is one half. Then we can compute the tau mass from the electron and muon, which is just the Koide prediction:

Using the tau prediction, we can go back and compute a tighter bound on the value of delta one and we get this interesting dimensionless number:

And finally, we can postulate these relationships and predict the neutrino masses to rather high precision:

$$\delta_0 = \delta_1 + \pi/12,$$
$$\mu_1/\mu_0 = 3^{11}.$$

$$\begin{aligned} m_\tau &= 1776.968921(158) \quad \text{MeV} \\ &= 1.907654627(46) \quad \text{AMU}. \end{aligned}$$

$$\begin{aligned} \delta_1 &= .22222204715(311) \quad \text{from MeV data} \\ &= .22222204717(48) \quad \text{from AMU data}. \end{aligned}$$

$$\begin{aligned} m_1 &= 0.000383462480(38) \quad \text{eV} \\ &= 0.4116639106(115) \times 10^{-12} \quad \text{AMU} \end{aligned}$$

$$\begin{aligned} m_2 &= 0.00891348724(79) \quad \text{eV} \\ &= 9.569022271(246) \times 10^{-12} \quad \text{AMU} \end{aligned}$$

$$\begin{aligned} m_3 &= 0.0507118044(45) \quad \text{eV} \\ &= 54.44136198(131) \times 10^{-12} \quad \text{AMU} \end{aligned}$$

If we multiply the tribimaximal mixing angle matrix by a matrix composed of the eigenvectors of the circulant matrix, the result is this guy, which is a 24th root of unity.

This has been an abbreviated description of the structure of the leptons. A complete description requires a quantum formalism that gives you matrices of Pauli algebra spin projection operators. But that sort of thing is equivalent to matrices of complex numbers, under certain assumptions.

I'm just starting work on the quarks. I think I can break the mesons down.

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & e^{+2i\pi/3} & e^{-2i\pi/3} \\ 1 & 1 & 1 \\ 1 & e^{-2i\pi/3} & e^{+2i\pi/3} \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{1/2} & 0 & -i\sqrt{1/2} \\ 0 & 1 & 0 \\ \sqrt{1/2} & 0 & i\sqrt{1/2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{1/24}$$

Now some theory.

We will begin with pure density matrices for spin one half. These are spin projection operators. I will use capital letters where the letter designates the spin direction.

Let's think about products of these operators that begin and end with the same operator, say Z. No matter what stuff I put in between the result has to be be a complex multiple of Z (maybe zero).

Complex multiples of Z commute with complex multiples of Z, so these types of products form a copy of the complex numbers.

$$R^2 = R.$$

$$ZRGBGBRZ = \eta\, Z.$$

where $\eta$ is a complex number.

Let me define bras and kets this way:

Now I just showed that stuff that begins and ends with Z all commute, so this is just like the bras and kets of the usual way of doing quantum mechanics.

By using density matrices, we eliminated the global U(1) gauge freedom normally present in spin one half wave functions. But we can go back to the bra and ket form of quantum mechanics any time we want by choosing a specific direction and making that our "Z".

Another way of explaining this is that we have geometrized the global U(1) gauge symmetry. It is no longer a mystery. We know exactly what it is. It's a preferred direction in space that you choose in order to convert a density matrix to bra ket form.

Now the standard model is built on global gauge symmetries that are a heck of a lot more complicated than the U(1) symmetry. How can we use this trick to geometrize these gauges too?

$$|R\rangle \;=\; RZ,$$
$$\langle G| \;=\; ZG$$

$$\langle G|R\rangle = ZGRZ,$$

In the 1950s, Schwinger developed a very elegant version of Quantum mechanics called the "Measurement algebra". In it, he uses the idea of a generalized Stern-Gerlach filter. This is like a piece of equipment that allows certain particle types and or orientations to pass and absorbs all others. Zero means a beam stop. One means an unimpeded beam. Multiplication means feeding the output of one filter into the input of another. Addition means allowing particles to pass through either of two filters.

Now measuring a particle twice is the same as measuring it once, so the measurements of fundamental particles, for example an electron with spin up, square just like density matrices.

The only difference with density matrices is that we can make filters that, for example, pass electrons and reject neutrinos, well at least theoreticians can.

Now to geometrize the Schwinger measurement algebra, we have to choose a geometric algebra. Since the usual density matrices for spin one half particles uses the Pauli algebra, it's natural to generalize this to a more complicated Geometric algebra. The Dirac algebra is an obvious choice.

If we assume some hidden dimensions, then the associated Clifford algebra becomes bigger and the geometry of the fermions (or the geometry of their preons) becomes more complicated. Back in 2004 I realized that this gave a geometric way to count the number of hidden dimensions. That paper was also rejected by arXiv after a physicist asked me to publish it.

The cube I showed in the first part of this lecture comes from these assumptions. The technical reason for choosing a cube is that this is the form of the spectral decomposition of a Clifford algebra. Scary stuff.

$$X_e \, X_e = X_e$$